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Loday, Jean-Louis (F-STRAS-I)
Cup-product for Leibniz cohomology and dual Leibniz algebras.
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Leibniz cohomology $HL^*(\mathfrak{g})$ of a Lie algebra $\mathfrak{g}$ is defined like the classical Lie algebra cohomology, but with the nth tensor product $\mathfrak{g}^{\otimes n}$ instead of the nth exterior product $\bigwedge^n \mathfrak{g}$. Leibniz cohomology is defined for a larger class of algebras: the Leibniz algebras. By definition a Leibniz algebra is a k-vector space $\mathfrak{g}$ endowed with a bilinear map $[\ ,\ ]\colon \mathfrak{g}\times\mathfrak{g}\to\mathfrak{g}$, satisfying the Leibniz identity

$$[x,[y,z]]=[[x,y],z]-[[x,z],y],\ \text{for all } x,y,z\in\mathfrak{g}.$$

Thus the Leibniz identity is equivalent to the Jacobi identity when the bracket is skew-symmetric.

The author constructs, by means of shuffles, a cup-product on the cohomology groups $HL^*(\mathfrak{g})$ of the Leibniz algebra $\mathfrak{g}$:

$$\cup\colon HL^p(\mathfrak{g})\times HL^q(\mathfrak{g})\to HL^{p+q}(\mathfrak{g}).$$

This product is neither associative nor commutative, but satisfies the formula

$$(f\cup g)\cup h=f\cup(g\cup h)+(-1)^{|h||g|}f\cup(h\cup g),$$

which is, in a certain sense, dual to the Leibniz identity.

The author describes some interesting features of this duality. *Elena Poletaeva*